Linear Algebra and its Applications 433 (2010) 877–892

Contents lists available at ScienceDirect

Linear Algebra and its Applications

journal homepage: www.elsevier.com/locate/laa

In memoriam Israel Gohberg August 23, 1928–October 12, 2009

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ARTICLE INFO

Article history:
Received 28 October 2009
Accepted 3 March 2010
Available online 2 April 2010

Submitted by H. Schneider

AMS classification:
01A-65
01A-70
01A-74
15-03
45-03
46-03
47-03
65-03
93-03

Keywords:
History of mathematics
Matrix and operator polynomials
Rational matrix functions
State space method in analysis

ABSTRACT

This obituary for Israel Gohberg consists of a general introduction, separate contributions of the six authors, all of whom worked closely with him, and a final note. The material gives an impression of the life of this great mathematician, of his monumental impact in the areas he worked in, of how he cooperated with colleagues, and of his ability to stimulate people in their mathematical activities.

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doi:10.1016/j.laa.2010.03.006

Systems theory
Realization
Factorization
Fredholm theory
Toeplitz operators
Wiener–Hopf integral equations
Singular integral equations
Partially specified matrices
Maximum entropy extensions
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Israel Gohberg

Introduction

Israel Gohberg was born on August 23, 1928 in Tarutino, Romania (at that time); he passed away on October 12, 2009 in Meir Hospital in Kfar Saba, a town bordering his hometown of Ra'anana, Israel, after close to three weeks of hospitalization. His mathematical legacy is monumental. More than 450 mathematical articles carry his name. He co-authored 26 books of which the last will appear posthumously this year. Several of the books originally written in Russian have been translated into other languages. Gohberg supervised 40 Ph.D. students. Many of them went on to build successful careers in mathematics; some succeeded in the business world. He was the founder (in 1978) and the Editor-in-Chief of the journal Integral Equations and Operator Theory and the book series Operator Theory: Advances and Applications. The journal has now more than 50 volumes and the book series over 200 titles. Together with J. William Helton he initiated a series of international workshops on operator theory and its applications (IWOTA), and he was the president of the corresponding steering committee. Up to now there have been 20 workshops carrying the IWOTA label, held in America, Europe, the Middle East, Africa, and Asia. All these tasks, he carried out until the end of his life. This in spite of the mounting health problems in the later years.

Israel Gohberg received numerous honors and awards. These included six honorary doctoral degrees from universities in Germany (Darmstadt), Austria (Viennna), Romania (Timisoara), Moldova (Chisinau and Balti), and Israel (Haifa, Technion). The prizes bestowed upon him were the Hans Schneider Prize

in Linear Algebra, the M.G. Krein Prize (Ukrainian Academy of Sciences), the Landau Prize and the Rothschild Prize (Israel), and the Alexander von Humboldt Prize (Germany). In 1970 he was elected a corresponding member of the Academy of Science of Moldova (removed 1974, reinstated 1996), and in 1985 he was elected Foreign Member of the Royal Netherlands Academy of Arts and Sciences. In 2009 he was elected SIAM Fellow.

At this place it is appropriate to recall what was said in the report (1993) on the first Hans Schneider Prize,[1] written by Daniel Hershkowitz from Haifa:

"*Israel Gohberg was born in 1928 and began his mathematical studies in Frunze (Kirgizia), and then in Kishinev (Moldava). Later, he became a close friend and collaborator of M.G. Krein in Odessa (Ukraine) and began a justly famous partnership that lasted for 24 years. In 1964 he obtained a doctoral degree at Moscow State University, and in 1970 he was elected a corresponding member of the Moldavian Academy. Since 1974 he has been a Professor at Tel Aviv University, Israel, and has been the incumbent of the Nathan and Lily Silver Chair since 1981. He has been a Foreign Member of the Royal Netherlands Academy of Arts and Science since 1985 and was awarded the Rothschild Prize in Mathematics in 1986. He has supervised more than 40 doctoral candidates.*[2] *Israel Gohberg has over 300 publications. He published seven books while in the Soviet Union, five of which have been translated into English, French or German. His books with Krein on Linear non-selfadjoint operators (translation by the AMS in 1969) and Volterra operators in Hilbert space (AMS 1970) have had profound impact on functional analysis and linear algebra. He has published seven more books in English since 1974. His book on Minimal Factorizations of Matrices and Operator Functions (1979) with Bart and Kaashoek is the seminal work on the subject and has led to great advances in the area in which many authors, not least Gohberg himself, have made significant contributions. The classification of pencils of matrices under equivalence achieved by Kronecker in the last century is still difficult today and the problem of extending it to general matrix polynomials is daunting. Gohberg and his many collaborators have made tremendous progress on this. His three important books with Lancaster and Rodman, Matrix Polynomials (1982), Matrices and Indefinite Inner Product Spaces (1983) and Invariant Subspaces of Matrices with Applications present accounts of more linear algebraic aspects of this work. His 1990 book with Ball and Rodman Interpolation of Rational Matrix Functions breaks significant new ground in this area. His contributions have covered most aspects of linear algebra. He has obtained important results on such diverse areas as completion problems, structured matrices, spectral theory, factorization and realization of matrix functions, canonical forms, perturbation theory, matrix functions and interpolation, complexity, and systems theory. Because of the strength and interest of his results, he has greatly influenced the direction of the development of linear algebra.*"

Apart from this introduction and the final note, the present obituary consists of six pieces written by Harm Bart, Harry Dym, Rien Kaashoek, Peter Lancaster, Alexander Markus, and Leiba Rodman, people who all worked intensively with Israel Gohberg for an extended period of time. Together they give an impression of how it was to work with this great mathematician and charismatic personality.

Israel Gohberg: Architect of the state space method for problems in analysis

Harm Bart

On October 12, 2009 Israel Gohberg passed away. He had a profound influence on my life. The way we met and came to a long lasting cooperation has been described in the Birkhäuser book *Israel Gohberg and Friends; On the occasion of his 80th birthday*. To honor this great mathematician and remarkable person, I will complete the story told there by adding some mathematical content.

The work of Israel Gohberg covers a wide area. It is monumental, both in quality and quantity. And this fact is even more striking when one takes into account his inspiring influence on others who expanded on themes that he initiated. So in trying to convey an impression of Israel's contribution to mathematics one has to make a choice. For me this means that I will write here about the project I actually worked on with him and Rien Kaashoek: *the state space method for problems in analysis*.

[1] Awarded to Shmuel Friedland, Israel Gohberg and Miroslav Fiedler. The Prize was presented to Israel Gohberg at the Fourth ILAS Conference (Rotterdam, 1994).

[2] In 1993, the year of the report, the number was actually 38.

It started in the fall of 1976. I had just returned to the VU University in Amsterdam from a one year stay at the University of Maryland at College Park. It happened in the office of Rien Kaashoek, my former Ph.D. supervisor. Israel and Rien were about to embark on a certain research project and they asked me to join them. Israel indicated the direction to be followed and I decided to accept the offer, not knowing then how decisive an impact this would have on my mathematical life for many years to come.

What was it all about? Israel started out by asking if I knew about the theory of characteristic operator functions as initiated by M.S. Livšic in the fifties. I did not, and he began to explain the basics of the theory taking the book *Triangular and Jordan representations of linear operators* by M.S. Brodskii as a point of reference. The characteristic operator function appearing there has the form

$$W(\lambda) = I_Y + C(\lambda I_X - A)^{-1}B, \tag{1}$$

where Y and X are (complex) Hilbert spaces, I_X and I_Y are the identity operators on X and Y, respectively, $A : X \to X$, $B : Y \to X$, $C : X \to Y$ are bounded linear operators, and $BC = A - A^*$ with A^* the Hilbert space adjoint of A. Under natural minimality conditions on the representation (1), the (closed) invariant subspaces of A correspond to certain factorizations of the analytic operator function W. The result in question has non-trivial applications. It can be used, for instance, to prove that the Volterra integral operator

$$A : L_2[0, 1] \to L_2[0, 1], \qquad Af(t) = i\int_0^t f(s)ds$$

is unicellular. In fact, its (closed) invariant subspaces are the invariant subspaces M_α consisting of all $f \in L_2[0, 1]$ with f vanishing (almost everywhere) on the interval $[0, \alpha]$.

Next Israel pointed out a surprising connection with results on monic operator polynomials that had been obtained by himself, Lancaster and Rodman.[3] The point was the following. Suppose L is a monic operator polynomial, say

$$L(\lambda) = \lambda^l I_Y + \lambda^{l-1}A_{l-1} + \cdots + \lambda A_1 + A_0.$$

Then there exist operators $T : Y^l \to Y^l$, $R : Y \to Y^l$ and $Q : Y^l \to Y$ such that, besides some other conditions that are satisfied,

$$L(\lambda)^{-1} = Q(\lambda I_{Y^l} - T)^{-1}R.$$

Here too, there is a close connection between factorization and invariant subspaces. In fact there is a one-to-one relationship between the factorizations of L into monic factors and invariant subspaces of T of a specific type. Working out the details led to the first Bart–Gohberg–Kaashoek paper *Operator Polynomials as Inverses of Characteristic Functions*. Israel considered it of such significance that he put it as the opening paper in the then newly founded journal Integral Equations and Operator Theory.

The significance of the paper was more in the point of view than in the complexity of the arguments which consisted mainly in clothing the pertinent results of Gohberg, Lancaster and Rodman in a different garment. But the point of view was fruitful indeed. It led us to considering factorizations of functions which can be written in the form (1) without a priori conditions on the "coefficients" A, B and C (except, of course, that the right hand side makes sense). An expression of this type is called a realization and there is a strong connection with mathematical systems theory. In fact, (in the matrix case) the right hand side of (1) is the transfer function of the linear time invariant system

$$\begin{cases} x'(t) = Ax(t) + Bu(t), \\ y(t) = Cx(t) + Du(t), \quad t \geqslant 0, \\ x(0) = 0, \end{cases}$$

which can be obtained by taking Laplace transforms. Another source of inspiration was the work by L.A. Sakhnovic. It gave us the idea that factorizations of functions of the general type (1) might correspond, not to invariant subspaces of the single operator A, but to pairs of invariant subspaces, one for A and the other for $A^\times = A - BC$.

[3] See the book I. Gohberg, P. Lancaster, L. Rodman, Matrix polynomials, Academic Press, 1982; re-publication SIAM, 2009.

The pieces ultimately fell into place in the following geometric factorization principle:

Factorization principle. *Let the operator function W be given by the realization*

$$W(\lambda) = I_Y + C(\lambda I_X - A)^{-1}B,$$

let M be an invariant subspace for A, and let $M^\times$ be an invariant subspace for $A^\times = A - BC$. Assume the two subspaces match, that is, $X = M\dot{+}M^\times$, write Π for the projection of X along M onto $M^\times$, and introduce

$$W_1(\lambda) = I_Y + C(\lambda I_X - A)^{-1}(I_X - \Pi)B,$$
$$W_2(\lambda) = I_Y + C\Pi(\lambda I_X - A)^{-1}B.$$

Then $W(\lambda) = W_1(\lambda)W_2(\lambda)$.

In fact, if working with different realizations of W is allowed, all possible factorizations of W (having the value I_Y at infinity) can be obtained in this way.

As we discovered during a miniconference held in February 1978 (one day at the VU University in Amsterdam and a second day at the Technical University in Delft) the result had been obtained independently by Paul Van Dooren who was, at that time, a Ph.D. student of Patrick Dewilde. To avoid fruitless priority claims, it was decided to combine the findings in a four authors paper Bart–Gohberg–Kaashoek–Van Dooren.[4]

The factorization principle as presented above is a rough version of the full result in question. There are other ways to formulate it, for instance making use of Riccati equations or in terms of direct sum decompositions of operators. The latter, in particular, sheds more light on the domain on which the factorization holds. Finally, the factorization results (of the Brodskii type and those on monic polynomials) referred to earlier fit into the more general framework of the factorization principle. In the Brodskii case this is easy: if M is an invariant subspace for A and $A - A^* = BC$, then $M^\perp$ is an invariant subspace for $A^* = A - BC = A^*$ and $X = M\dot{+}M^\perp$. The factorization result for monic polynomials can be recovered too, but this is somewhat more involved and requires a suitable Möbius transformation.

Israel Gohberg understood at once the potential of the Factorization Principle and led the way in applying it to a variety of problems. In doing so, basically two possibilities offer themselves. The first is the choice of the matching subspaces, the second the choice of the realization. As to the first of these options, working with spectral subspaces M and $M^\times$ turned out to be especially useful in dealing with Wiener–Hopf factorization, a kind of factorization that comes up in considering Wiener–Hopf integral equations, block Toeplitz matrices and singular integral equations. As for the second option, the so called minimal realizations are the appropriate ones for dealing with minimal factorizations. These are factorizations of rational matrix functions where no pole zero cancellation occurs, and they play a role in the theory of electrical networks. In all these applications, a striking feature is the high degree of explicitness: "up to the end" as Israel used to say.

The problem how to compute minimal factorizations in practice has been discussed in the four authors paper mentioned above. The accuracy of the method presented there depends on the minimal angle between the subspaces M and $M^\times$. Further analysis led to a detailed investigation of stability issues for factorizations and invariant subspaces.

Initially the theory was mainly concerned with realizations involving operators on finite dimensional spaces (so matrices) only. But quickly also infinite dimensional situations entered the picture. The first such case involved the transport equation as it occurs in a branch of mathematical physics dealing with the analysis of equations that describe the migration of particles in a medium. For example, a flow of electrons through a metal strip or radiative transfer in a stellar atmosphere. Later, in dealing with certain non-rational matrix functions, we had to work in an infinite dimensional context too. In some cases, it was even necessary to admit unbounded operators. Of course, in order to develop a sensible theory it was necessary to impose additional constraints. Out of this necessity, concepts like exponentially dichotomous operator and their associated strongly continuous bisemigroups emerged.

[4] Factorizations of transfer functions, SIAM J. Control Opt. 18 (1980) 675–696.

Israel Gohberg was gifted with an extraordinary mathematical intuition. Many times his remarks – sometimes made in passing – sparked off a complete new project. One such a case influenced my personal development tremendously. It was an off hand observation made one afternoon in the early 1980s when he, Rien and I sat together. The observation was about minimal factorization of a function into McMillan degree one factors, i.e., factors of the form $I_m + (\lambda - \lambda_0)^{-1}R$ with R a rank one $m \times m$ matrix. Such factorizations (which do not always exist) are said to be complete. In the first Bart–Gohberg–Kaashoek book *Minimal factorization of matrix and operator functions* (Birkhäuser, OT 1 (1979), the factorization principle is used to prove that a rational matrix function admits complete factorization provided it has poles of order one only. Israel remarked that the key fact behind this result is really the following: *If A and Z are $m \times m$ matrices, and A is diagonalizable, then there exists an invertible $m \times m$ matrix S such that $S^{-1}AS$ is upper triangular and $S^{-1}ZS$ is lower triangular*. The remark led to a series of publications on reduction of pairs of matrices to complementary forms (as it was named), this time written by me in cooperation with colleagues from the Erasmus University in Rotterdam, my home base since 1984. The work culminated in the discovery of a connection between factorization into elementary factors and the two machine flow shop problem from operations research.[5] Besides being a great mathematician himself, Israel was a source of inspiration to others.

My own involvement in developing the state space method for analysis problems, in direct cooperation with Israel Gohberg and Rien Kaashoek, lasted for about ten years: from 1976 to (approximately) 1986. After that, for another period of about ten years, I continued this line of research, but now with colleagues from the Erasmus University in Rotterdam. The contacts with Israel remained however, though they were somewhat less intense than during the time I was still at the VU University in Amsterdam. We saw each other at seminars, workshops, conferences and, occasionally, in the family atmosphere at home. In the last ten years, the situation changed again. The close scientific contacts with Israel and Rien were re-established, now in the context of the writing of two monographs,[6] Israel's last two books. In this, André Ran was asked to participate as a fourth author, especially because of his expertise in factorizations having special symmetry features. The four of us also wrote a paper[7] about something we discovered in working on the books, namely that the key tools from the state space method for problems in analysis[8] can be traced back in a unifying way to the notion of Schur complements.

Good and strong memories have resulted from all of this. The title of my contribution to the book *Israel Gohberg and Friends; On the occasion of his 80th birthday* that appeared in 2009 is "Israel Gohberg: a teacher and a friend". My mathematical life has been influenced deeply by Israel as a teacher; through his passing I lost a dear friend.

ISRAEL GOHBERG Z"L

Harry Dym

Israel Gohberg immigrated to Israel with his wife Bella, their two daughters, Zvia and Yanina, and his mother Clara towards the end of July 1974. Shortly thereafter he was offered positions at a number of Israeli universities, including the Weizmann Institute of Science, which is a Research Institute with graduate students. Motivated in large part by a desire to interact with students at an earlier stage of their education, Israel accepted the offer from Tel Aviv University. The Weizmann Institute then offered him a half-time position in the hope that he would eventually leave Tel Aviv and come to the Institute on a full-time basis. He accepted the half-time position, but never left Tel Aviv.

[5] For details and references, see Part III in H. Bart, I. Gohberg, M.A. Kaashoek, A.C.M. Ran, Factorization of matrix and operator functions: the state space method, Birkhäuser, OT 178, 2008.

[6] Factorization of Matrix and Operator Functions: The State Space Method, Birkhäuser, OT 178, 2008, and A State Space Approach to Canonical Factorization: Convolution Equations and Mathematical Systems, Birkhäuser, OT 200, 2010.

[7] H. Bart, I. Gohberg, M.A. Kaashoek, A.C.M. Ran, Schur complements and realizations of transfer functions, Linear Algebra Appl. 399 (2005) 203–224.

[8] For a recent review of the state space method, see I. Gohberg, M.A. Kaashoek, State space methods for analysis problems involving rational matrix functions, in: G. Picci, D.S. Gilliam (Eds.), Dynamical Systems, Control, Coding, Computer Vision, Birkhäuser, 1999, pp. 93–110, and the references given there.

Some time in 1975 Israel started coming to the Institute twice a week on a regular basis. At that time, the Department of Pure Mathematics (as it was then called) was very small and had more office space than full-time faculty. Israel was given an office two doors down from mine. In contrast to the other members of the department, Israel kept his office door open. Passing it from time to time I sensed that he would welcome the opportunity to interact with some of the "natives" and initiated contact. I do not remember precisely when we started working together on a regular basis; probably in the Fall of 1975.

Israel would come to the Institute for a full day on Sunday; He would lecture from 11:00 to 13:00. Then we would go to lunch. After lunch we would usually sit together until about 18:30. He would also spend a second afternoon at the Institute, mostly sitting with me. This pattern continued for almost 10 years and ended when the Institute cancelled all part-time positions because of budgetary problems. During this period we wrote a number of papers together that focused primarily on extensions of assorted classes of partially specified matrix valued functions, with special interest in *maximum entropy extensions*. A few papers down the line, we realized that many of these extension problems could be incorporated in a general scheme that got called *the band method*. This approach was later extended further by Israel together with some of his colleagues in Amsterdam.

Looking back, I think that Israel's open office door was in many ways characteristic of his personality: open, friendly, optimistic and positive. I never heard him disparage the work of other mathematicians.

In general, Mathematics is not an overly sociable business. Progress is typically achieved by very small groups; individuals working alone, or with one and occasionally two collaborators. Among Israel's many achievements was the bringing of a sense of community to many of the individuals working in and around operator theory. He accomplished this through personal contacts, extensive travel and collaborations. Apart from his full-time position at Tel Aviv and half-time position at the Weizmann Institute, Israel had many other homes; most notably the VU University of Amsterdam, the University of Calgary and the University of Maryland.

Israel was instrumental in initiating the IWOTA (International Workshop on Operator Theory and its Applications) conferences which meet at least once every two years from 1981. With the help of generous support from the Silver family he inaugurated the bi-annual Toeplitz Lecture series at Tel Aviv University, followed by a humongous party at the Gohberg home, catered by the Gohberg women: mother, wife, sister and daughters. The first of these featured Peter Lax and Ciprian Foias as guest lecturers. All three, host and guests, escapees from "unfriendly" regimes. Israel was also involved in the organization of a number of Oberwolfach meetings which served to bring together many of his far flung friends and collaborators and were in some sense like family reunions. I think that the first of these was in the Fall of 1976. An unanticipated bonus for Israel was a side trip to a Michelin three star restaurant in Illehausern, France; his first and probably his last.

Israel is well known as the founder and editor of the journal *Integral Equations and Operator Theory*, which expanded from a quarterly at its inception to its current monthly status, and the book series *Operator Theory: Advances and Applications*, which will publish its 200th volume in the very near future. Two other journals devoted to these topics that started at roughly the same time as IEOT folded after a few years.

One of Israel's favorite questions at the end of a seminar or lecture was: What can you say in the finite dimensional case? In my younger years I used to think that this was just a nice way to eliminate the embarrassment of the speaker in the all too often deathly silence at the end of the lecture when the speaker turns to the audience for questions. It was only much later that I realized that this was really a very serious question and the answers to it are often both elegant and illuminating.

In the past several years Israel has been undergoing dialysis, which in the most recent past entailed five weekly four hour sessions. Nevertheless he continued to travel, lecture, collaborate on articles and books and to plan for the future. Even when he was hospitalized towards the end of September 2009, he asked me to extend his regrets that he was unable to participate in the meeting in Chemnitz, but that he hoped to be able to come to Berlin. Unfortunately, that was not to be.

I would like to close with two stories about Israel that I have related on three previous happier occasions: Israel's 60th, 70th and 80th birthday meetings. They have been printed in the Conference Proceedings of those meetings and are reprinted (in triplicate) in the recent Birkhäuser volume *Israel Gohberg and Friends*. However, they are too good not to share.

When Israel first met my youngest son Michael, he asked him "How many children are there in your class?" "Forty one," was the reply. "Wonderful," said Israel, "so many friends."

One Sunday morning Israel came to the Institute much later than usual. It turned out that he had had a traffic accident enroute. Another car had hit his car from the rear. As accidents go, it was relatively minor, but still the expense and inconvenience was far from negligible. After settling in, he called his wife to tell her what happened. "But why did he do this to you?" was her immediate wifely response. "Belochka," said Israel, ever so gently, "this question you must put to him, not to me."

In the *Sayings of the Fathers*, a tractate in one of the six books of the Mishnah that were compiled some 1800 years ago, there is a saying that translates roughly to: *Acquire a teacher and acquire a friend.* To so many of us, Israel Gohberg was both. *Yehi Zichro Boruch*.

Israel Gohberg: Innovative and inspiring, reminiscences from Amsterdam

Rien Kaashoek

The first time Israel Gohberg visited Amsterdam was in December 1975, about a year and a half after he had emigrated from the Soviet Union to Israel. His second visit, which included a bicycle trip to the famous flower exhibition at the Keukenhof, took place in early Spring 1976. We had started a collaboration in the first two months of 1975, when both of us were in College Park, Israel as a guest of his friend Seymour Goldberg, and I on sabbatical leave working with David Lay. Israel's first two visits to the Netherlands were in preparation of a longer stay of six months as guest professor at the mathematics department of the VU in the Autumn semester of 1976. That Autumn semester was an enormous success, mathematically as well otherwise.

At the end of 1976 all parties involved agreed that each year Gohberg would come to Amsterdam for two or three short periods, ranging from two to eight weeks. The contract would be for intervals of three years. In 1983 the three year contract was replaced by an appointment as a part time professor which lasted until 1998 when Gohberg retired from the VU at the age of 70. As emeritus he continued to visit the VU each year for a few months until 2006. At that time he had already been a dialysis patient for several years, and he had a number of bypasses. We were well aware of the fact that he was vulnerable, but we also knew his strong will-power and we shared in his unflagging optimism. He was mathematically active up to the very end. For many of us his passing away in October 12, 2009 still came rather unexpectedly.

The last time when we were together was in May 2009, first at the Haifa Matrix Conference where he gave a beautiful lecture on "Matrix Polynomials" honoring his former student and co-author Leiba Rodman, on the occasion of Leiba's 60th birthday, and then later at Tel Aviv discussing plans for papers and the second Bart–Gohberg–Kaashoek–Ran book.

When Gohberg in 1974, almost 46 years old, left the Soviet Union and immigrated to Israel, he already had a great international status. Together with Mark Krein he was one of the pioneers of the new Fredholm operator theory. Their famous theorem (1958) which describes the Fredholm index of a block Toeplitz operator with a continuous symbol in geometric terms as the winding number around zero of the determinant of this symbol was a great result. Nowadays it is viewed as an early predecessor of the Atyah–Singer index theorem from the late 1960s. The two books on the theory of non-selfadjoint operators, which Gohberg and Krein wrote together, are of world class. Gohberg's work (together with Nahum Krupnik) on singular integral equations and, in particular, the use in this context of the Gelfand theory of Banach algebras, was extremely and is still a source of inspiration. His work with Jürgen Leiterer, at that time a student from the DDR, was used extensively later, among others in linear transport theory. The formulas of Gohberg–Sementsul and Gohberg–Heinig for the inverse of Toeplitz and block Toeplitz matrices and their continuous analogues received much attention. In due time they became the corner stones of a theory of structured matrices and operators which had a strong impact on numerical linear algebra.

In operator theory it is very common to analyze concrete classes of integral and differential equations in terms of analytic functions which are in a natural way related to the equations involved. Analyzing Toeplitz equations in terms of the associated symbol is an example where such a reduction

is very useful. In 1976 when Gohberg was in Amsterdam, he suggested as a next step a second reduction, which would become one of the highlights in the work he, Harm Bart and I were going to do jointly. His suggestion was, on the one hand, inspired by the theory of characteristic functions for operators close to selfadjoint or unitary operators, and on the other hand by the spectral analysis of matrix and operator polynomials, work he had been doing just after his emigration to Israel with Peter Lancaster and Leiba Rodman.

The new step Gohberg had in mind was based on the observation that the analytic functions appearing in operator theory often can be analyzed in terms of three or four operators which are often much simpler than the original operator. It uses the idea of state space realization which is common in mathematical system and control theory, and it views the analytic functions involved as transfer functions of linear input–output systems. For instance, consider a system of Wiener–Hopf integral equations with a rational matrix symbol. Then the symbol has no pole at infinity and a classical realization theorem of Kalman tells us that the symbol can be written in the form

$$W(\lambda) = D + C(\lambda I - A)^{-1}B$$

with A, B, C and D matrices. In this way the problem of solving the Wiener–Hopf integral equation, which involves an operator on an infinite dimensional space, reduces to a linear algebra problem. Nowadays we refer to this approach as the *state space method*. It has been used to solve problems in analysis, ranging from infinite systems of linear Toeplitz equations and transport problems in astrophysics to canonical differential systems and nonlinear integrable partial differential equations.

The state space method opened a new chapter in the relation between analysis and linear algebra, which Gohberg and his co-workers at different institutions, not only at Amsterdam, have exploited and developed in various directions. To mention just one example, Leonid Lerer and his student I. Haimovici employed the state space method to present an abstract scheme for defining Bezout operators, in which they needed to write certain analytic matrix functions as transfer functions of possibly infinite dimensional systems. The four of us used this result in 2005 to explicitly express the null space of the Gohberg–Heinig Bezout integral operator in terms of the common eigenvectors and common Jordan chains of the two underlying entire functions. The work done in Amsterdam resulted in three books, the first written in 1979 with Gohberg, Bart and myself as authors, and the second and third written by the three of us with André Ran as the fourth author. The second became available in the fall of 2007 and the third book will appear posthumously this year.

The reconstruction of matrices and operators on the basis of partial information is another theme that was very prominent in the work of Gohberg after his emigration. As for the state space method, this second theme was highly motivated by system and control theory problems. It dealt with a wide variety of problems from complex analysis and matrix and operator theory. It includes the so-called *band method*, which originated from his work in the late seventies with Harry Dym at the Weizmann Institute, and which was developed further in Amsterdam in joint work of Gohberg with Hugo Woerdeman and myself. The band method is an abstract scheme which allows one to deal with positive and contractive completion problems from one point of view, and which presents a natural strategy to solve such problems by reduction to linear equations. It led to beautiful and easy to handle explicit formulas for the solutions of various completion and interpolation problems, which were worked out in detail for the Carathéodory problem, the Nehari problem and its four block generalization; time-varying (non-stationary) analogues of these problems were also solved. The connection between maximum entropy solutions and central interpolants or central completions was one the other highlights in this area.

This second theme also includes a study of the similarity invariants of various partially given matrices and operators. Freek van Schagen participated actively in this research. It led to remarkable applications to eigenvalue completion problems, to stabilization problems in mathematical system theory and to problems of Wiener–Hopf factorization. The three of us wrote a book on this topic which appeared in 1995. After the book was published we discovered yet another beautiful connection, namely between the theory of operator blocks presented in our book and the 1970 paper of Gelfand and Ponomarev on the classification of quadruples of subspaces in finite-dimensional vector space.

The work that Ciprian Foias, Art Frazho, Israel Gohberg and I did jointly in the period 1995–1998 is connected with both themes referred to above. It resulted in the book entitled *Metric constrained interpolation, commutant lifting and systems*, published in 1998 with a special "golden cover" as volume

100 of the OT series. This book deals with stationary and nonstationary metric constrained interpolation problems, in finite or infinite dimensions, and it offers a unified approach based on commutant lifting from operator theory and the state space method. The three-chain completion theorem, a nonstationary version of the commutant lifting theorem, is one of the highlights.

Joint work with Israel Gohberg was always a pleasure and very satisfying. His charismatic personality, his personal charm, his great mind and great memory, his wonderful sense of humor, his principles and rules for good cooperation which dated from the time he worked with Mark Krein, his talent as a story teller, the unexpected mathematical gems he presented in lectures and discussions, it all helped him to create an inspiring and relaxed working environment. Gohberg enjoyed being in Amsterdam. He loved to go to the fish market in the Albert Cuypstraat. He knew the best restaurants. He liked one of the Chinese restaurants in particular, where he would often entertain his guests. This became known as *Gohberg's restaurant*. When I once told Israel that in the Netherlands Amsterdam is known as Mokum (Yiddish for a good place to be), he immediately recognized the Hebrew origin: Makom, and a Makom it has been for him. And for us: thanks to him, it became a better place to live.

Gohberg's mathematical legacy is enormous. More than 450 mathematical articles carry his name. He co-authored 26 books of which the last will appear posthumously this year. Of the 26 books seven were mainly written in Amsterdam and five are co-authored by his late friend Seymour Goldberg. He was the founder (in 1978) and the editor of the journal Integral Equations and Operator Theory and the book series Operator Theory: Advances and Applications. The journal has now more than 50 volumes and the book series over 200 titles. Together with Bill Helton, he founded a series of international workshops on operator theory and its applications (IWOTA), and he was the president of the IWOTA Steering Committee. Up to now there have been 20 workshops carrying the IWOTA label, held in America, Europe, the Middle East, Africa, and Asia, with three in the Netherlands. All these tasks he carried out until the end of his life. A very remarkable achievement.

In the Bible, when one of the kings of Israel has died, it is often written: "The rest of his acts and every thing he did, is that not written in the book of the Chronicles of the kings of Israel?" In this vein I want to conclude with a reference to the book *Israel Gohberg and Friends* which was published in 2008 by Birkhäuser Verlag on the occasion of Gohberg's 80th birthday. His exceptional course of life, his 40 Ph.D. students including five from the VU, his six honorary doctorates, his election in 1985 as a member of the Royal Dutch Academy, in the seat that became vacant after Mark Kac passed away, the Hans Schneider prize in Linear Algebra and other awards and grants he received, his mathematical legacy, and his influence on the work and lives of co-workers and friends are all recorded in that book. And not only that. *Israel Gohberg and Friends* also contains articles written by Gohberg himself about his vision on mathematics, and sixteen articles written or co-authored by him about his relation with some of his colleagues and friends. Altogether it presents a fascinating picture of a great mathematician and exceptional friend. We remember him with gratitude.

In the Netherlands and in many countries all over the world Israel Gohberg will be missed dearly. Our sincere sympathy is with his family, in particular, with Bella his wife since 1956, who in the last 10 years or so always joined him on his foreign journeys. As Israel mentioned many times: without her he would not have been able to make these trips, and we would not have been able to enjoy his company.

The Gohberg–Lancaster partnership

Peter Lancaster

Soon after his immigration to Israel in 1974, Israel Gohberg accepted an invitation to visit the University of Toronto – extended by Israel Halperin. I learned of this and was able to invite him to visit the University of Calgary. Thus, his first visit to Calgary was in 1975, when he lectured on operator theory. At the time, I knew of the Krein school of mathematics and, in particular, was familiar with the Gohberg/Krein monograph on linear nonselfadjoint operators (published by the AMS in 1969). In the preceding years I had published in mechanics, linear algebra, approximation theory and numerical analysis, and had given graduate courses in operator theory. We communicated well together from

the beginning, and it became clear in our second meeting (a few months later in Dundee) that we had good grounds for collaboration. At the time I was working on a pair of papers which were submitted to LAA in March, 1976, and published in 1977. We discussed this work, and this led to discussion of canonical structures for matrix polynomials, related inverse problems, and applications to differential and difference equations.

At this time I.G. had a talented Ph.D. student, Leiba Rodman, who was working in the same area, and a three-way collaboration evolved. In the following years I arranged for I.G. to come to Calgary as a Visiting Scholar. These visits would last for some months. He had furnished accommodation with a suburban family, and I provided a bicycle to get to and from the university. He seemed to enjoy his accommodation and returned to it more than once. And, of course, he got to know me and my family well. No doubt he missed his own family, but we were able to develop a warm family environment which, together with our work together, persuaded him to return to Calgary. Our first three-author paper appeared (appropriately) in LAA in 1978; the first of several more on matrix polynomials, perturbation theory, indefinite scalar products, systems theory, and so on. Leiba Rodman spent the years 1978–1980 in Calgary as a Post-Doctoral Fellow, and these were the formative years for our first three three-author monographs (appearing in 1982, 1983, and 1986). Fortunately, I.G. shared a love of the outdoors with my family. Some of our happiest memories of him are connected with hiking, mushrooming, and quiet times in the Rocky Mountains and Alberta foothills.

Israel Gohberg and Peter Lancaster, Canadian Rocky Mountains, 1975.

In later years I was I.G.'s guest in Tel Aviv for some extended working visits. My wife, Edna, was with me for some of these. We were warmly received by the Gohberg family, and I got to know more of the Gohberg "school" of mathematicians - which generated other collaborations (with V. Matsaev and A.S. Markus, for example) and post-doctoral appointments in Calgary. I.G.'s ability to collaborate

freely with a great variety of people depended on creating an easy atmosphere of mutual respect. He was always the "ideas" man: proposing one line of attack or another – and with an uncanny sense of which argument would work when examined in detail.

Need I say, getting to know I.G. was a turning point in my life, personally and mathematically. He led the way to scholarship, deeper mathematics, and unassuming, sincere friendship.

Israel Gohberg: my teacher and friend

Alexander Markus

Israel Gohberg passed away on October 12, 2009. This happened almost exactly 20 years after the day when his teacher Mark Krein died, October 17, 1989.

The contribution of I.G. to linear algebra, operator theory and integral equations is so vast and profound, and his influence on the development of international scientific relations and the editorial and publishing trade in these areas so strong, that today it is too early to sum them up. The goal of the current remarks is very modest. I will try to re-create his image as I remember him.

In the academic year 1950–1951 I started to study mathematics at Kishinev University. After several months I heard about one mathematics student with extraordinary talent. He had already published several papers in the leading Soviet scientific journal "Doklady". This was I.G. I wanted to meet him immediately, but for a rather long time I felt shy. The end of the academic year, when he had to leave the university, was approaching, and I decided that it was now or never. On May 29, 1951 I came up to I.G. in the university library and asked him to give me 5 min. (All details were kept in my notes of the same day.) We went to the courtyard, and the conversation started. It continued two hours.

I introduced myself and rather shyly asked some questions. I wanted to know (neither more nor less!) how to score success in mathematics. He told me that it is absolutely insufficient to obtain the higher marks for exams (probably he knew that I got them). "You have to search for the meaning of each proof you are studying, whether all conditions are essential? What about the inverse theorem? Which analogues and generalizations are possible?" He advised me to study some new area which was not yet in a syllabus. "For example, functional analysis", he said. "By the way, the first Russian book in this domain, namely by Lyusternik and Sobolev, will be published soon. After you read this book, we can discuss some concrete problems." This conversation gave me wings!

A month later I.G. graduated from the University and left Kishinev. In the next three years our meetings were accidental. In 1954 I ventured to ask him for some problem. Very soon he proposed to me an interesting problem about semi-Fredholm analytic operator functions. We started to discuss this problem. Incidentally we came to another problem and solved it. As a result, our joint paper was published in "Doklady" at the end of 1955 (this was my first publication). I succeeded in resolving the initial problem only in 1957, and this was one of the main results of my Ph.D. thesis.

After Stalin's death the so-called thaw began in USSR, and some positive changes in our life occurred. After my graduation in 1955 I was immediately accepted to graduate studies. This was impossible for I.G. in 1951, although his achievements then were much higher that mine in 1955.

In 1959 important changes in our lives took place. I.G. and I started to work in the Moldavian branch of the Academy of USSR. This became possible since this Moldavian branch was in the process of being transformed into the Moldavian Academy, and in this institution all main sciences had to be represented. Of course, it was very important that the thaw continued for the time being.

The next 15 years were the nicest in my mathematical life. Our task was to sit in an office in the Institute of Mathematics and to do research. I.G. as the head of our department (the Department of Functional Analysis) gave us full freedom in the choice of problems and encouraged any initiative, although, when necessary he offered concrete problems and helped in their solutions. It was very important that we could immediately know his new results and inform him of our results and difficulties. The seminar in functional analysis organized and headed by I.G. brought together many people who worked in this area, or were interested to do this. One of the most active participants of the seminar was Nahum Krupnik from the University of Kishinev. Many outstanding mathematicians from the USSR and from abroad gave talks at the seminar.

Israel Gohberg; the picture was taken in 1967 soon after he became professor.

I.G. by himself has written at length about the Department of Functional Analysis in his "Mathematical Tales" (see the fine book *Israel Gohberg and Friends*, pp. 22–26). I want to quote a passage from there and to add some comments. I.G. wrote: "Matsaev and Markus were customers of J. Leiterer and myself. We proved for them theorems on factorization of operator functions, and they produced very interesting results in the theory of operator polynomials and operator functions." For my friend Volodya Matsaev and for me the possibility to discuss our results and our difficulties with such an expert in factorization as I.G. was priceless. The results by I.G. and Jürgen Leiterer, V. Matsaev and myself mentioned here are presented in my book on the spectral theory of operator polynomials (Sections 24–27).

Kishinev, Department of Functional Analysis.
From the left to the right: I. Gohberg, M.S. Budjanu, I. Feldman, A. Markus, A.A. Sementsul.
Photograph taken in December 1966.

In 1973 I.G. and his family decided to leave the Soviet Union for Israel. Exit visa applications had to be accompanied by various papers. One of them was a reference from the employer. Usually this demanded a meeting of all colleagues where they condemned the applicant's behavior (as a rule, in a gross tone). Such a meeting was arranged in our Institute of Mathematics on November 26, 1973. We knew that this would be a shameful performance, but it was impossible to miss it since this was during our working hours. On this day, one of the active communist party members came to my office and told me that I had to speak at the meeting. When I refused he said the same to my friend Israel Feldman sitting in the same office, and obtained the same answer. This meeting was one of the most painful experiences of my life. Before the meeting I.G. took a sedative, and he behaved with dignity. There was no lack of speakers but no one from our department spoke.

After I.G. left the USSR our situation in the Institute sharply worsened. The Department of Functional Analysis was closed and its members were dispersed among other departments. It became very complicated to keep scientific contacts with I.G. It was quite risky to use the postal service for this purpose. Preprints in our Institute were not issued. On the other hand, I.G. constantly sent me preprints and reprints. I read them carefully and saw how successfully he worked and how the range of his interests broadened. Now it also included various problems from matrix theory, matrix polynomials and system theory.

In 1978, I got a preprint of a paper by I.G. and Rien Kaashoek. It contained a very interesting conjecture concerning the behaviour of the sizes of the blocks in a Jordan matrix under small perturbation. Soon after this my student E. Parilis and I confirmed this conjecture. We wrote a paper and submitted it to the journal "Matem. Issled." (By the way, this journal was organized by I.G. in Kishinev in 1966.) This journal was little-known abroad. But it was for us absolutely impossible to send manuscripts to foreign journals, and the publication in Moscow journals (like "Math. Sbornik") was rather complicated (see "Israel Gohberg and Friends", p. 26). But in this case we had big difficulties even with the publication in "Matem. Issled." I was informed that our paper could be published, but only with no mention of I.G. Finally the following very strange reference was written in the paper: "Professor M.A. Kaashoek and his colleagues suggested that…" For a long time after this I burned with shame remembering this story. Fortunately, Chandler Davis was interested in our paper and translated it into English. He was in touch with me and made all needed corrections in the translation. This translation was published in LAA. By the way, almost the same results were obtained by other methods at the same time by H. den Boer and G.Ph.A. Thijsse.

I arrived in Israel only in January 1990, more than 15 years after I.G. I started to participate in his seminar in Tel-Aviv University and endeavored to maintain contact with him by phone or by e-mail. But, unlike the Kishinev years, we could not meet every day to discuss in detail various mathematical (and other) problems. Nevertheless, when I had some essential difficulties, I called on him for advice, and he always helped me. In these years I could see the fruits of his activity in research, in editing, publishing and organization of conferences. They were huge and outstanding.

He continued to work almost with the same intensity after his retirement, up to his last hospitalization in September 2009. He did this in spite of painful illness and the time taken up in treatment. He was optimistic by nature, and also a man of great wit and humor. These virtues helped him in the struggle. He would joke even in grave states. The following story happened after his first heart attack in the end of 1968. It was known (or at least it was conventional at that time) that patients in those conditions were afraid to be alone. His family (all of them doctors!) and his friends tried to prevent him from being alone. Soon he took notice of this and said smiling: "I know that I should be afraid. But in spite of my respect for medical science, this is not the case with me!"

I.G. was able to relax a tension with a joke. I will give only one example. In Kishinev I acquired a habit of having a nap after lunch. Our lunch break continued one hour, and my lunch break was in reality twice as long. On the other hand, we often were working in our offices late into the night. Once I.G. asked me to come to the office on Sunday to help him with some bureaucratic work (something like "Working plan" or "Report on the fulfillment of the plan"). I had other plans for this day, and answered: "Why should I work on Sunday?!" He smiled and said: "OK, you are not obliged. But then you shouldn't sleep in the working time". We both laughed, and the question was settled.

After the death of Israel Gohberg there remain his books and papers, his collaborators and followers, the journal IEOT, the book series OTAA, and the IWOTA conferences, all founded by him. But our world became much poorer without him. May his memory be blessed.

Israel Gohberg: teacher and mentor

Leiba Rodman

As one of the 40 Ph.D. students of Israel Gohberg, I will focus my remarks on Israel Gohberg – teacher and mentor.

Every working mathematician knows that it is not easy to come up with a good research problem or project. It should be nontrivial but doable, of interest to other mathematicians, justified in some sense by the history of mathematical development and/or bona fide applications, and of course it should involve novel ideas and approaches – which at the start of the project are not evident at all, and one has to have faith that useful ideas will be developed in the course of the research. Israel Gohberg had a truly remarkable ability to formulate viable research projects, drawing on his vast experience and deep and broad scholarship. He was also generous with open problems, and on many occasions would share them within his circle of collaborators and friends. When collaborating with Gohberg, often the issue was to select one (or more) of many potential projects to work on rather than to come up with a project. Needless to say, this was a boon to a novice (and not only novice) mathematician who came into Gohberg's orbit, and his doctoral students and postdoctoral mentorees benefitted the most. Just follow the direction of research pointed out by Israel Gohberg, and if you are diligent and persistent the results will come.

My first consequential meeting with Israel Gohberg took place at Tel Aviv University in Spring of 1974, when as an aspiring doctoral student, I was looking for a potential supervisor. Remarkably, after the very first meeting Gohberg agreed to guide me, without really knowing much about my background, abilities, etc., and without going through my personal file, list of courses taken, or grades. Then my background in operator theory was very basic. The choice of topic suitable for me was not easy. However, at that time Israel Gohberg was working with Georg Heinig (1947–2005), one of his former Ph.D. students, on matrix polynomials and related topics. So, he suggested to me the topic of matrix polynomials, which turned out to be remarkably productive, in collaboration with Peter Lancaster. I recall that my first assignment given by Gohberg was an easy, almost trivial, calculation. Gradually, the assignments became more and more demanding, and after a short while I was working on bona fide research problems.

It is commonly known that writing mathematical texts, especially original research articles, is notoriously difficult, not the least because one has to explain precise mathematical constructions using imprecise human language. We all know many examples of brilliant mathematicians who simply cannot write, to the detriment of our community. Gohberg's approach to this problem, when working with colleagues and especially when supervising graduate students and mentoring postdocs, was uncompromising: High quality of writing was emphasized along with the high quality of mathematics. There was never any doubt if a major re-writing should be done in order to achieve a modest improvement, or to present ideas and proofs in a more understandable reader friendly form. Interestingly, after a series of modest improvements each of which could be debatable and perhaps even rejected, the text was dramatically improved.

My experience as a Ph.D. student under the guidance of Israel Gohberg, and later as his junior collaborator, illuminates many characteristics of his teaching and mentoring ways that I am sure were also experienced by his other students and mentorees: willingness to give students a chance to prove themselves, basic optimism that things will work out well despite temporary difficulties and setbacks, a keen sense of what is possible and desirable, and above all a singular ability to guide a student or mentoree toward achieving the maximum of his or her ability and potential.

Final note

The Birkhäuser book *Israel Gohberg and Friends* published in 2008 on the occasion of Israel Gohberg's 80th birthday contains a wealth of biographical material on Israel Gohberg, an almost complete

bibliography of his publications, short pieces written by his colleagues, friends, and family, as well as by Gohberg himself, and several addresses he gave at a variety of occasions. The material will be complemented by the article "Speeches and reminiscences" in volume 1 of another Birkhäuser book, namely the Proceedings of the IWOTA 2008 Conference at Williamsburg, USA, a meeting celebrating the 80th birthday of Israel Gohberg.[9]

One contribution to *Israel Gohberg and Friends* deserves to be singled out. It is the story by his two daughters Zvia and Yanina on how their father, when they were young, tried to give them an impression of the wonderful world of mathematics. Among other things he told them about his work with Boltyanski on the minimal number of light sources needed to illuminate a convex body. Not only is the story moving, it also sheds light on lesser known aspects of Israel Gohberg's mathematical activities, and on how his daughters, in spite of not being mathematicians, came to appreciate their father's mathematics.

On October 12, 2009, the world lost a great man and scholar. He was an example and a source of inspiration to many; many, including the six authors of this obituary. will remember him with gratitude. Being close to him, we saw how strongly he was supported by his wife Bella, his two daughters Zvia and Yanina, and his sister Fanny. Our thanks go to them too.

[9] Topics in Operator Theory Volume 1: Operators, Matrices and Analytic functions, Volume 2: Systems and Mathematical Physics, J.A. Ball, V. Bolotnikov, J.W. Helton, L. Rodman, I.M. Spitkovsky (Eds.), Birkhäuser, OT 202–203, 2010.